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Advances in Supersonic Configuration Design Methods

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A linearized panel method program developed for the design and analysis of wing-body configurations at supersonic speeds (Woodward 1) has been modified to permit the user to specify the loading for an arbitrary region of the wing. The remaining portion of the wing is then optimized on the basis of the prescribed loading. This insures that the final shape will produce the minimum drag possible within the constraints. In addition, the design feature has been incorporated into an improved panel method supersonic analysis program (Woodward II). Since this code has the capability of representing bodies of noncircular cross section, it is expected to prove useful in the design of aircraft configurations with arbitrarily shaped fuselages. Comparisons are presented with other known minimum drag solutions which demonstrate the validity of both new techniques.

Nomenclature

$A_{(\ )}$	= panel area
$a_{(\ )}$	= aerodynamic coefficient
$b_{(\ )}$	= aerodynamic coefficient
$C_{(\ )}$	= nondimensional coefficient
D	= pressure drag
F	= auxiliary function
L	= lift
M	= moment, Mach number
$N_{(\ )}$	= number of panels
$n_{(\ )}$	= unit normal velocity
$p_{(\ )}$	= panel pressure
x	= freestream direction
z_c	= camber line ordinate
α	= angle of attack
λ	= LaGrange multiplier
$2y/b$	= span station

Subscripts

B	= body
D	= drag
i	= influenced panel number
j	= influencing panel number
L	= lift
P	= pressure
R	= reduced influence coefficient
W	= wing
WB	= body on wing
BWS	= wing on body sources

Introduction

SINCE its initial development by Woodward et al.,[1] the linearized panel method program (Woodward I) has been widely used as a tool for the analysis and design of supersonic configurations. Despite the development of other methods,[2-5] it is still in common use.[6-9] More recently, Woodward[10] developed an improved method for the analysis of supersonic configurations (Woodward II).

Neither the Woodward I nor II code entirely fulfills the needs of an aircraft designer. The Woodward I code can be used to optimize the wing camber for minimum drag at a prescribed lift and moment, either for the wing alone or in the presence of a fuselage and nacelles. However, the program imposes no geometric constraints; it will occasionally produce unrealistic cambers and pressures. This is usually overcome by the simple expedient of fairing in a reasonable camber shape. Obviously, there is no guarantee that such a modified wing is close to the optimum. Furthermore, the program cannot handle arbitrary bodies.

The Woodward II program appears to be superior to the Woodward I program because of its capability of representing arbitrary body shapes, rounded wing leading edges, and wing interference effects in the presence of body closure. However, it lacks the design option.

A method of imposing constraints in the design option of the Woodward I program by specifying the pressures on an arbitrary number of wing panels is described. This permits the user to specify the loading for an arbitrary region of the wing and have the remaining portion optimized for minimum drag, insuring that the final shape will produce the minimum drag possible within the constraints.

The implementation of the design option in the Woodward II program is also described.

Method of Approach

The basic linear theory of aerodynamic influence coefficients is described in detail in Ref. 1. The highlights of the derivation of the wing optimization matrix are repeated herein.

$$L = \text{lift} = \sum_{i=i}^{N_W} A_i p_{W_i}$$

$$D = \text{drag} = \sum_{i=i}^{N_W} A_i \left(\frac{dz_c}{dx}\right)_i p_{W_i}$$

$$M = \text{moment} = \sum_{i=i}^{N_W} A_i (x_i - \bar{x}) p_{W_i}$$

Introducing LaGrange multipliers λ_1 and λ_2, a function F is set up which will equal the wing drag when the wing lift and pitching moment are equal to their constrained values:

$$F = D + \lambda_1 (L - \bar{L}) + \lambda_2 (M - \bar{M})$$

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The condition for minimum drag may be written:

$$\frac{\partial F}{\partial p_{W_i}} = \frac{\partial D}{\partial p_{W_i}} + \lambda_1 \frac{\partial L}{\partial p_{W_i}} + \lambda_2 \frac{\partial M}{\partial p_{W_i}} = 0$$

$$\frac{\partial F}{\partial \lambda_1} = L - \bar{L} = 0 \qquad \frac{\partial F}{\partial \lambda_2} = M - \bar{M} = 0$$

The camber slope for panel i is given by:

$$\left(\frac{dz_c}{dx}\right)_i = n_{WB_i} - \sum_{j=1}^{N_B} b_{ij} n_{BWS_j} + \sum_{j=1}^{N_W} a_{R_{ij}} p_{W_j}$$

The equation for the wing drag now becomes:

$$D = \sum_{i=1}^{N_W} D_i = + \sum_{i=1}^{N_W} A_i p_{W_i} \left(n_{WB_i} - \sum_{j=1}^{N_B} b_{ij} n_{BWS_j} + \sum_{j=1}^{N_W} a_{R_{ij}} p_{W_j} \right)$$

Therefore,

$$\frac{\partial D}{\partial p_{W_i}} = + A_i \left(n_{WB_i} - \sum_{j=1}^{N_B} b_{ij} n_{BWS_j} \right) + \sum_{j=1}^{N_W} (A_i a_{R_{ij}} + A_j a_{R_{ji}}) p_{W_j}$$

The condition for minimum drag may now be written as:

$$\sum_{j=1}^{N_W} (A_i a_{R_{ij}} + A_j a_{R_{ji}}) p_{W_j} + \lambda_1 A_i + \lambda_2 A_i (x_i - \bar{x}) = A_i \left(n_{WB_i} - \sum_{j=1}^{N_B} b_{ij} n_{BWS_j} \right)$$

$$\sum_{j=1}^{N_W} A_i p_{W_i} (x_i - \bar{x}) = \bar{M}$$

$$\sum_{j=1}^{N_W} A_i p_{W_i} = \bar{L}$$

Constrained Drag Minimization

One possible way of applying additional constraints would be to set up an additional set of LaGrange multipliers. The method used to provide additional constraints in the Woodward I program is somewhat simpler and more direct. It was assumed that any $N1$ pressures in the drag minimization matrix were known. The remaining unknown pressures were determined by partitioning the matrix:

$$\left[\begin{array}{c|c} (A_1 a_{R_{11}} + A_1 a_{R_{11}}) \cdots & +A_1 (x_1 - \bar{x}) A_1 \\ (A_2 a_{R_{21}} + A_1 a_{R_{12}}) \cdots & +A_2 (x_2 - \bar{x}) A_2 \\ \vdots & \\ (A_{11}) & (A_{12}) \\ \hline (A_{21}) & (A_{22}) \\ (A_N a_{R_{N1}} + A_1 a_{R_{1N}} & +A_N (x_N - \bar{x}) A_N \\ A_1 \qquad +A_2 & 0 \quad 0 \\ (x_1 - \bar{x}) A_1 (x_2 - \bar{x}) A_2 & 0 \quad 0 \end{array}\right] \left[\begin{array}{c} p_{W_1} \\ \\ (P_1) \\ p_{W_{N1}} \\ \hline (P_2) \\ p_{W_N} \\ \lambda_1 \\ \lambda_2 \end{array}\right] = \left[\begin{array}{c} A_1 \left(n_{WB_1} - \sum_{j=1}^{N} b_{1_j} n_{BWS_j} \right) \\ \vdots \\ (B_1) \\ \hline (B_2) \\ A_N \left(n_{WB_N} - \sum_{j=1}^{N} b_{N_j} n_{BWS_j} \right) \\ \bar{L} \\ \bar{M} \end{array}\right]$$

In terms of the partitions, the matrix becomes

$$[A_{11}][P_1] + [A_{12}][P_2] = [B_1]$$

$$[A_{21}][P_1] + [A_{22}][P_2] = [B_2]$$

The solution for the unknown pressures was, therefore, found from

$$[P_2] = [A_{22}]^{-1} \{ [B_2] - [A_{21}][P_1] \}$$

This required only minor programming modifications to the drag minimization matrix present in the Woodward I program.

Implementation of the Design Option in the Woodward II Program

The design option was incorporated in the Woodward II[10] program using a procedure similar to that employed in the Woodward I[1] program. The only difference in the drag minimization matrix was the absence of the normal velocity on the wing due to body sources and doublets. Certain modifications had to be made to the code to enable it to operate in the design mode. Since the Woodward II program assigns an additional control point at the trailing edge for wings with supersonic trailing edges, an additional row and column of influence coefficients are introduced into the drag minimization matrix. This required the definition of an additional panel area for each row of wing panels. This was done by assigning a reduced panel area for each control point. This procedure produced reasonable results for subsonic leading edges (Figs. 1a and 1b).

For supersonic leading edges, the Woodward II designs appeared unrealistic. However, for these cases, the Woodward II program did not function properly in the analysis mode (Fig. 2). Careful examination of the pressures in these cases (Fig. 3) suggested the following possible explanations:

1) Linearly varying vortex singularities are a poor approximation for supersonic leading edges, since ahead of the Mach wave the pressures are constant, and

2) Putting the control point at the leading edge for all panels did not give the proper upstream influence for panels close to the trailing edge.

The linearly varying vortex singularities in Woodward II were replaced by constant ones. Furthermore, the additional control point at the trailing edge was eliminated. The control

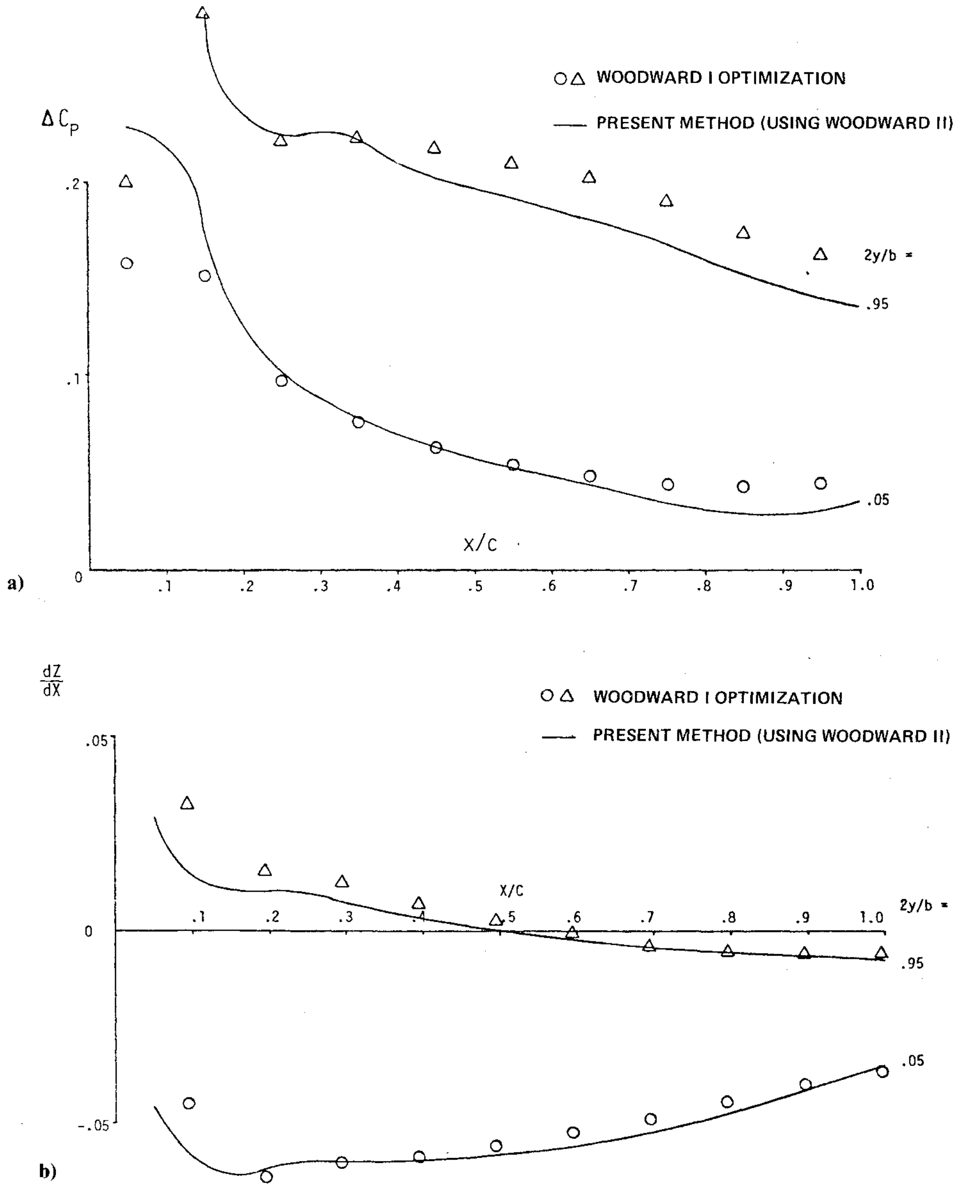

Fig. 1 a) Delta wing pressure difference; b) Delta wing camber slopes. $M = 1.414$; $\beta \cot\Lambda = 0.833$; $C_L = 0.1$.

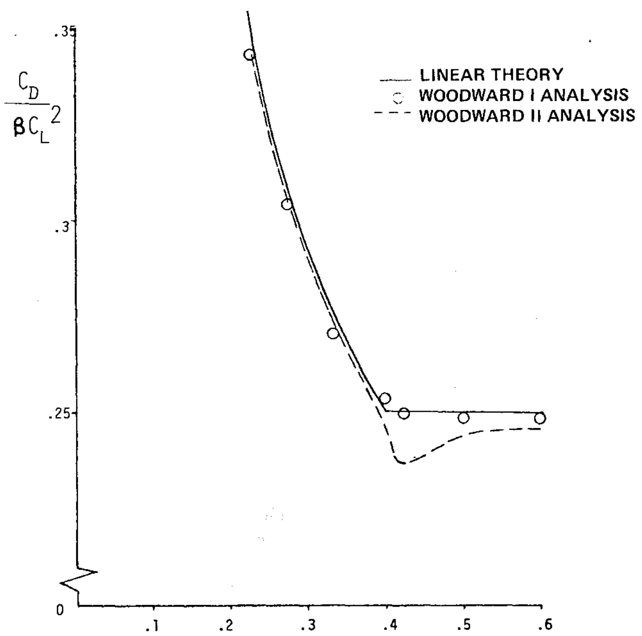

Fig. 2 Flat plate delta wing.

point has been fixed at the 95% chord of the panel, with the exception of panels which lie in front of the Mach wave from the root. For these panels, the control point remains at the leading edge (in the analysis mode only). In the design mode, the control point is always at 95% chord.

In essence, this reduced the Woodward II program to the Woodward I program for wing-alone configurations with subsonic leading edges (Fig. 4). For supersonic leading edges, the modifications improved the correlation with linear theory (Fig. 3). At this time, the constrained optimization feature has not been incorporated in the Woodward II design program.

Results

The constrained optimization program was used to design a delta wing with zero leading-edge loading. As may be seen from Figs. 5a and 5b, the constrained and unconstrained versions of the Woodward I program give practically the same result. This is not surprising, since experience has shown that Woodward I designs tend to produce wings with unloaded leading edges, satisfying the criteria for minimum drag advanced by Yoshihara.[11,12] Although Jones[13,14] and Cohen[15] indicated that it was possible to achieve optimum wing warps which had unconstrained leading-edge forces, the constrained designs appear preferable since they would minimize separation effects at the leading edge. It is interesting to note

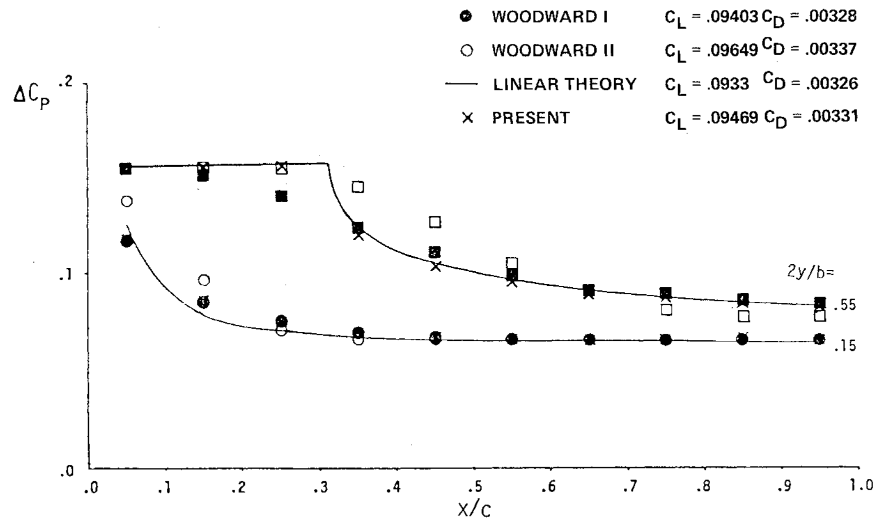

Fig. 3 Flat delta wing $\beta \cot \Lambda = 1.247$.

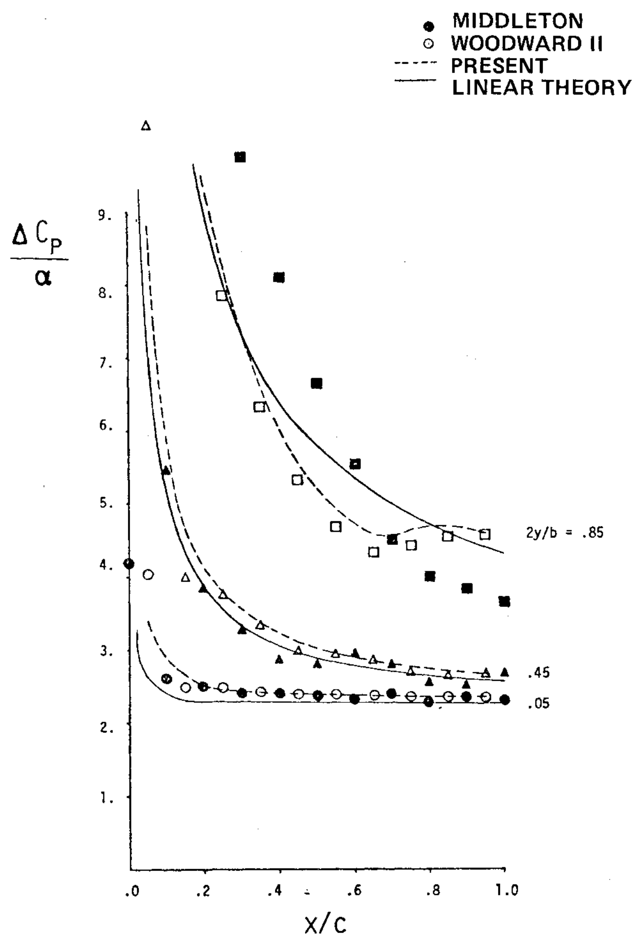

Fig. 4 Flat delta wing. $M = 1.414$; $\beta \cot \Lambda = 0.833$.

that while Yoshihara purposely selected optimized designs with finite leading-edge loadings, the Woodward I program automatically produces such designs, even though there is no mechanism in the program to insure this.

For this particular case, the major advantage of the constrained version of the program is its capability of defining the camber shape at the leading edge itself. This was not practical to do with the original Woodward I program, since it would have required specifying panels with very small chords ($x/c = 0.01$) along the leading edge. Such nonuniform panel spacing is known to produce undesirable oscillations in wing optimization calculations. The constrained version of the program does not experience this difficulty since the leading-edge panels are eliminated from the inversion of the drag minimization matrix, since the pressure on these panels is prescribed.

The Woodward I program calculates the panel slopes at the panel control points (0.95 of the panel chord). These slopes have to be extrapolated to the panel centroids for the optimized wings to be analyzed. Since the drag is calculated by multiplying the panel slopes by the panel pressure difference, this extrapolation can have a large effect on the drag calculation. This is strikingly illustrated in Fig. 6, where a wing optimized for minimum drag at a C_L of 0.1, when analyzed, achieved the minimum drag condition at a C_L 10% lower. When this wing was reoptimized with the constrained optimization program to define the leading-edge camber, Woodward I analysis of the resulting camber shape showed that the wing actually achieved minimum drag at the design point. Note that the wing appears to be fairly insensitive to the design C_L.

For the preceding two cases, the constrained optimization program required 110 wing panels (compared to the 100 panels distributed evenly spanwise and chordwise in the Woodward I program) to define the camber shape at the leading edge. In all of the following comparisons, both codes were run with 100 wing panels, distributed evenly chordwise and spanwise.

Using the method of Ref. 3, Mack[16] designed a series of wings for minimum drag. These wings had to be modified in the region of the wing root. Analysis of the wings that were tested indicated that there was a drag penalty associated with this modification. As another application of the constrained optimization program, these wings were redesigned under the constraint that the pressures in the modified regions be those due to the modified cambers (those pressures were calculated with the Woodward I analysis program to avoid differences between codes being mixed up with differences in the designs). As may be seen in Fig. 7, there is only a slight change in the camber slopes outside the constrained region. However, almost half of the theoretical decrement of modifying the optimized shape is recovered (Fig. 8). It must be emphasized that it would be preferable, in this case, if the camber slopes, rather than the wing loadings, could be constrained; however, by specifying the pressures, the camber slopes were matched almost identically at the first try (Fig. 7).

A wing-body configuration was designed using the constrained optimization program with the wing root pressures

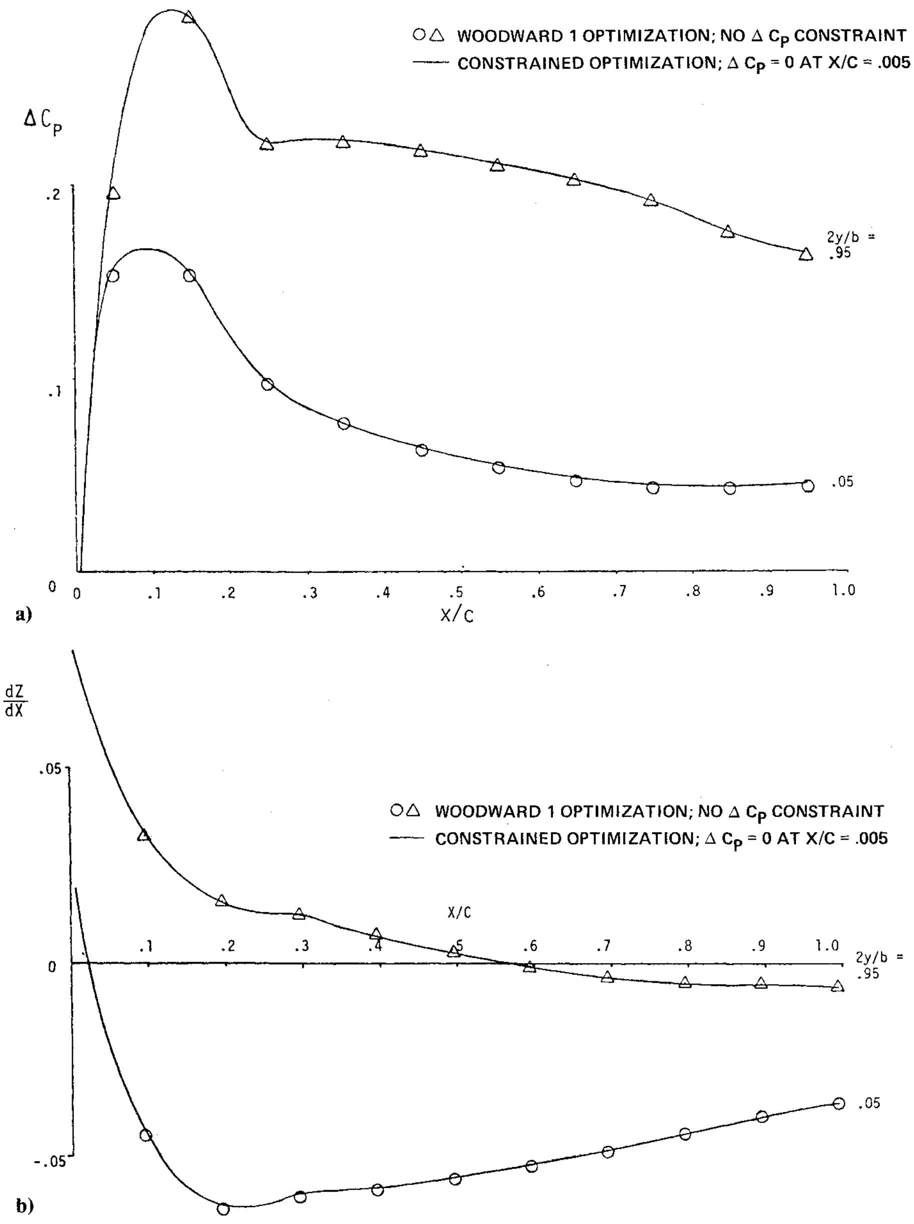

Fig. 5 a) Pressure difference; b) Camber slopes. Delta wing optimized for $\Delta C_p = 0$ at L.E. $M = 1.414$; β cot$\Lambda = 0.833$; $C_L = 0.1$; $T/C = 0$.

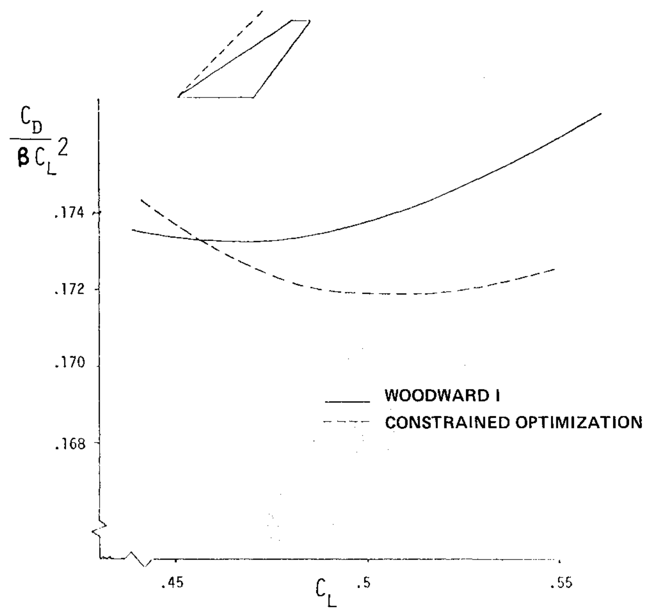

Fig. 6 Clipped arrow wing optimization. $M = 1.414$; $C_{L_{DES}} = 0.5$.

constrained to produce minimal camber change at the wing-body juncture. This was accomplished by prescribing root pressures which the Woodward I program predicted for this configuration when run with a flat wing. As may be seen from Fig. 9, nearly a constant camber shape was achieved. Furthermore, if the root camber alone had been modified in the same fashion without reoptimizing the rest of the wing, a 70% drag penalty would have resulted.

The Boeing wing body and the Ogive wing body, which were used to demonstrate the capabilities of the Woodward I[1] and Woodward II[10] codes, respectively, were next designed for minimum drag at a $C_L = 0.1$ with both programs. The Boeing wing body (Figs. 10a and 10b) appeared to carry more lift inboard and less outboard for the Woodward II design. For the Ogive wing body (Figs. 11a and 11b), the loading was relatively the same for both programs. The camber shape, however, was much smoother for the Woodward II design at the wing root. A possible explanation for this is that the Woodward II program does not contain the source and doublet terms on the right-hand side of the drag minimization matrix. Therefore, when the matrix is inverted and post-multiplied by the right-hand side, the only term appearing on the right-hand side is the design lift and moment. This might lead to a smoother solution.

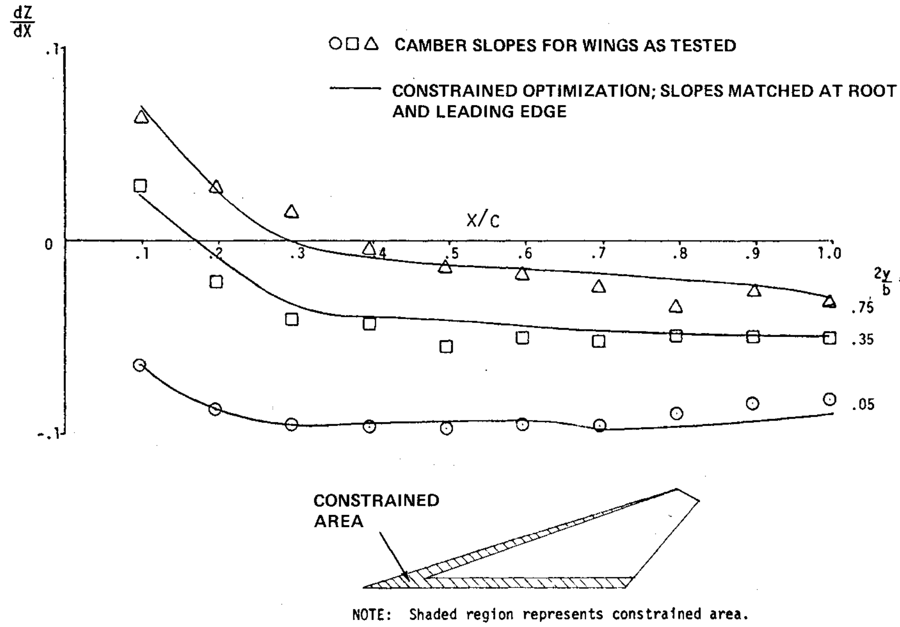

Fig. 7 Camber slopes. $M=2.6$; $\beta \cot \Lambda = 0.75$; $C_L = 0.08$; $T/C = 0$.

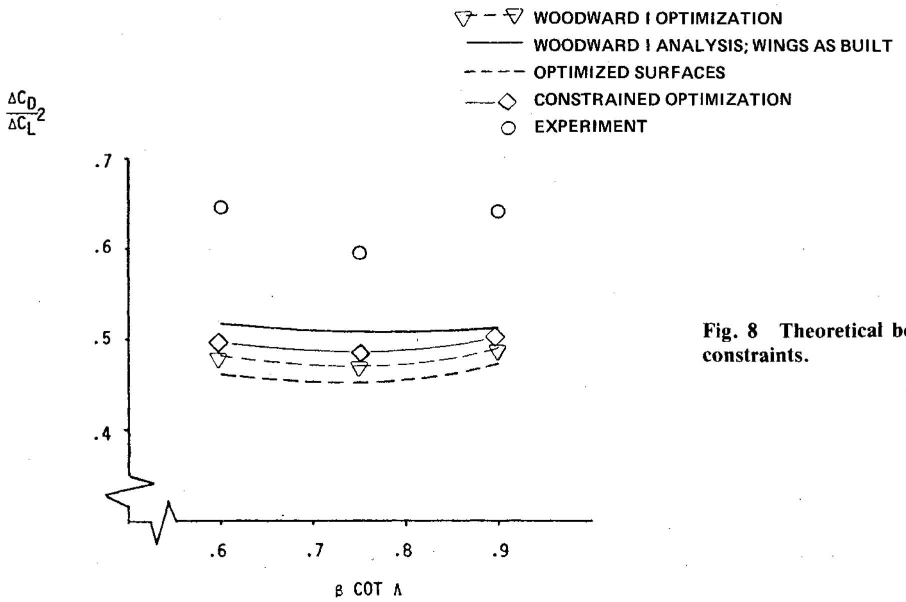

Fig. 8 Theoretical benefits of redesigning wings to match physical constraints.

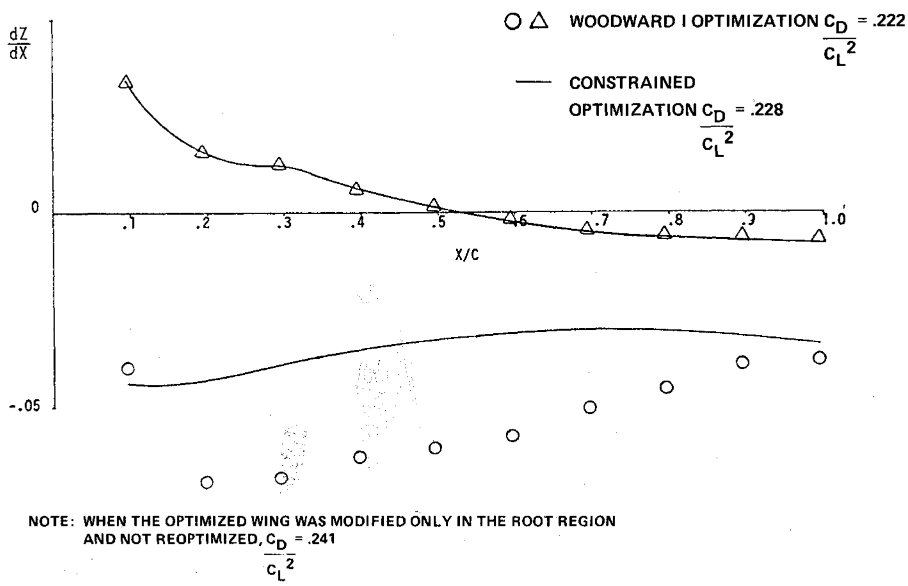

Fig. 9 Camber slopes. Wing optimized for small camber variation at wing body juncture.

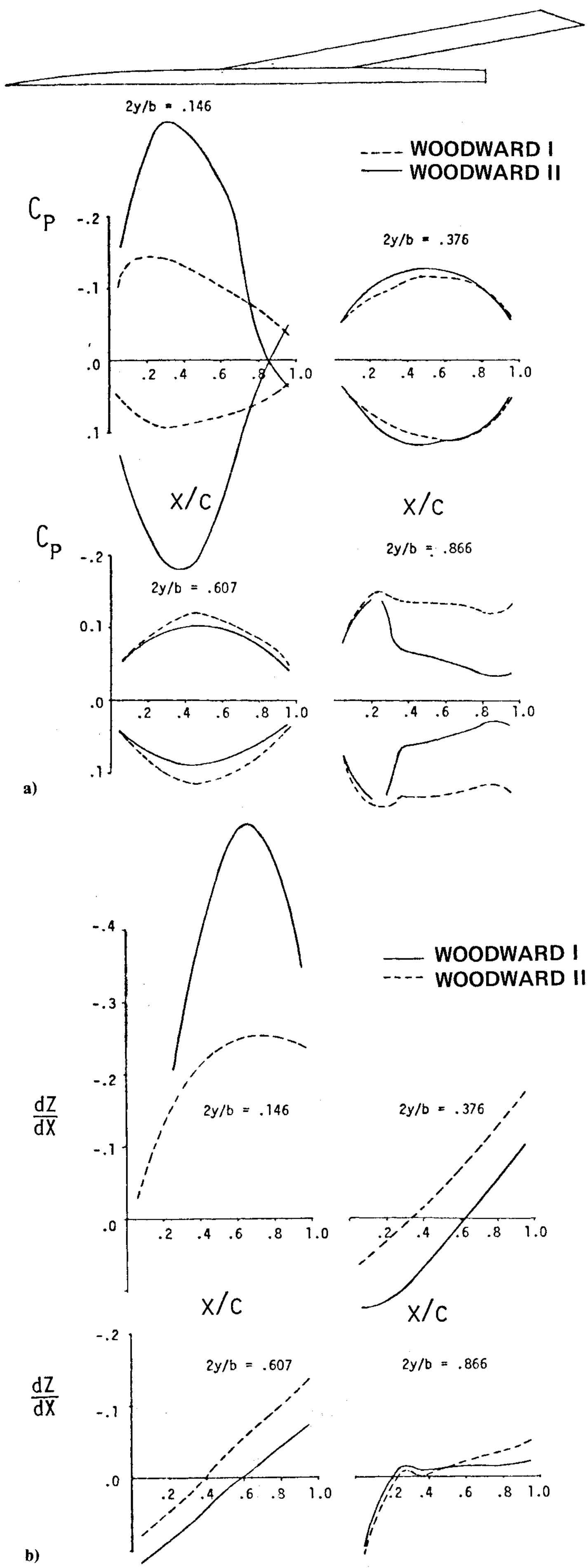

Fig. 10 TR-805 minimum drag.

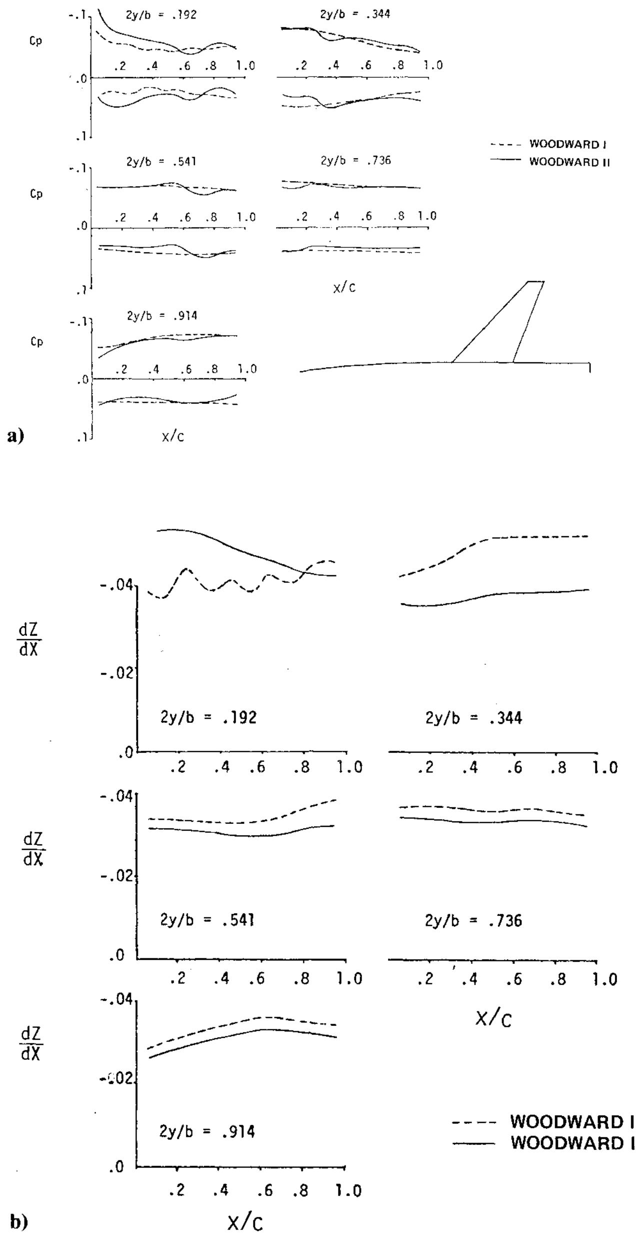

Fig. 11 Wing-body optimization.

Conclusions

Two new procedures for determining the wing camber surface for minimum drag have been developed. One involves the imposition of pressure constraints in the Woodward I program. This feature enables the user to tailor the wing design to satisfy any physical or geometric criteria deemed appropriate. The other is the implementation of the design option in the Woodward II program. This should prove useful in the design of aircraft with arbitrarily shaped fuselages, although the constrained optimization feature has not yet been incorporated in the Woodward II design code.

Both techniques are new approaches, although they incorporate ideas present in other supersonic design methods. Comparisons with established minimum drag solutions indicate the validity of the new methods. However, their value will not be determined until a supersonic aircraft is actually designed and tested with their aid.

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